Time-dependent one-dimensional spin-1 Ising system with weak coupling

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A modified version of Glauber's one-dimensional spin relaxation model is applied to a spin-1 Ising chain in order to study the time dependence of the system in the weak-coupling limit. The individual spin-1 Ising particles are assumed to interact with the heat bath, which causes them to change their states randomly in time. Coupling between the particles is introduced through the assumption that the transition probabilities for any one spin-1 Ising particle depends on the state of the neighboring spin-1 Ising particles. A special assumption about the rate constants is chosen such that the average values of the dipole moment will return to the equilibrium value. We establish the system of rate equations for average values of the dipole and quadruple moments, as well as their coupling. The Ising interaction between the spin-1 particles is assumed to be weak compared to the coupling with the heat reservoir. In this way we can terminate the hierarchy and solve the problem of a linear chain with periodic boundary conditions, using the Fourier transformation. The resulting secular equation determines two sets of relaxation times and two sets of eigenvectors. From this equation both relaxation times are determined by a perturbation method. [S1063-651X(97)12505-3]

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I. INTRODUCTION

In recent years, sustained research effort has been utilized in investigating equilibrium properties of spin-1 Ising systems (also known as the Blume-Emery-Griffiths model) in order to study the thermodynamical behavior of certain cooperative phenomena such as phase separation and superfluid ordering in He^3-He^4 mixtures [1], condensation and solidification of a simple fluid and as well as binary fluids [2], tricritical points in binary and ternary fluids [3], microemulsions [4], ordering in semiconductor alloys [5], electronic conduction models [6], magnetic materials [7], the reentrant phenomenon in phase diagrams [8], critical behavior and multicritical phase diagrams [9–12] and study of metastable and unstable states [13]. The above calculations were done by mean-field approximations [1–5,7,10], renormalization-group techniques [9], effective field theory [8], Monte Carlo methods [11], and cluster variation method [13,14] and its modified version [12]. Besides these methods, spin-1 Ising systems have been studied by other techniques such as series expansion methods [15], the Monte Carlo renormalization technique [16], the constant coupling approximation [17], and the transfer matrix method [18]. Some exact results for the two-dimensional honeycomb lattice have been obtained for a limited subspace of the bilinear, biquadratic, and crystal-field interaction parameters [19]. Thus, although the equilibrium properties of spin-1 Ising systems have been studied extensively, the dynamic behavior of spin-1 Ising systems has not been as thoroughly explored because dynamic models of cooperative phenomena are of a more speculative nature.

An early attempt to study the time-dependent one-dimensional spin-1 Ising system was made by Obokata [20]. He used the spin-1 Bethe method, but ignored the crystal field and subsequently extended it into a time-dependent model. He also obtained relaxation times and investigated the temperature dependence of the static reduced susceptibilities. Tanaka and Takahashi [21] studied a simple dynamic model of the spin-1 Ising system in the molecular field approximation and also obtained relaxation curves of order parameters. They showed that only one of the two relaxation times goes to infinity at the critical temperature. They did not consider the details of the interaction of the spin system with the heat bath. Batten and Lemberg [22] studied the dynamics of a spin-1 Ising system with the mean-field technique and investigated the relaxation of order parameters, but they did introduce the crystal field. Saito and Müller-Krumbhaar [23] also investigated the kinetics of a spin-1 antiferromagnetic Ising model using the time-dependent Ginzburg-Landau theory and applied it to crystal growth. Achiam [24] used the real-space renormalization-group approach to study the dynamic behavior of a spin-1 Ising system and found the dynamic exponents. We have also studied a number of nonequilibrium behaviors [25,26], in particular metastable and unstable states of order parameters of a spin-1 Ising system via the path probability method [27].

The purpose of the present paper is to study the time dependence of the one-dimensional spin-1 Ising system in the weak-coupling limit. This type of calculation was originally introduced by Glauber [28]. He studied the time-dependent statistic of the spin-$\frac{1}{2}$ Ising model for strong coupling. Meijer, Tanaka, and Barry [29] created the same problem in the weak-coupling limit in the sense that the external field is considered to be predominant.

The outline of the remaining part of this paper is as follows. In Sec. II the one-dimensional spin-1 Ising system is presented and the derivation of the basic master equation is given. The time dependence of the one-dimensional spin-1 Ising system is first studied for the uncoupled case and then for weak-coupling cases extensively in Secs. III and IV, respectively. A summary and the discussion of the results are given in Sec. V. Finally, the master equation for the pair distribution is given in the Appendix.

II. ONE-DIMENSIONAL SPIN-1 ISING SYSTEM AND DERIVATION OF THE MASTER EQUATION

The model we shall discuss is a stochastic one. The Ising spins S_k on N fixed locations whose total spin values equal 1 can attain the projected values $+1$, 0, and -1. The average value of S_k can be written as

$$\langle S_k\rangle=\sum_{S_k} S_k P(S_k), \tag{1}$$

where $S_k=-1,0,1$. Since the probalities are time dependent, due to the transitions among these three values the spin average is also a function of time. These transitions take place because of the interaction of the spins with a heat bath. On the other hand, the transition probabilities of the individual spins are assumed to depend on the momentary values of the neighboring spins and also the heat reservoir. Therefore, the statistical correlations arise between the values of neighboring spins.

We shall assume that these particles are arranged in an orderly spaced linear array, i.e., they form an N-particle chain. It is also assumed that individual spins in the chain are not totally independent stochastic functions.

The most simple Hamiltonian of the uncoupled spin-1 Ising system can be written as

$$\mathcal{H}(S_k)=-(\mu H_s S_k+\Delta S_k^2), \tag{2}$$

where $S_k=+1$, 0, or -1, which corresponds to the magnetization that is the excess of one orientation over the other orientation, also called the dipole moment, and S_k^2 takes only the values $+1$ and 0, which correspond to the quadrupole moment. H_s and Δ are the fields due to the dipole and quadrupole moments, respectively. Δ is also called the crystal field. We take the bilinear, biquadratic, and as well as odd interaction parameters to be zero because first we want to study the most simple case and then generalize to difficult cases. For the coupling case, the fields acting upon the kth spin will be

$$H_s=H_S^{(0)}+(S_{k-1}+S_{k+1})M \tag{3}$$

and

$$\Delta=\Delta^{(0)}+(S_{k-1}^2+S_{k+1}^2)D, \tag{4}$$

where M and D are the coupling constants corresponding to the dipole and the quadrupole moments, respectively.

A complete statistical description of this time-dependent one-dimensional spin-1 Ising system would consist of the knowledge of the probability function $P(S_k,t)$. The time dependence of this probability function is assumed to be governed by the master equation. The master equation describes the interaction between the spins and the heat bath and can be written as

$$\frac{dP(S_k,t)}{dt}=\sum_{S_{k-1},S_k',S_{k+1}} \omega(S_k;S_k',S_{k-1},S_{k+1})\times P(S_{k-1},S_k',S_{k+1}). \tag{5}$$

The rate of change will be influenced by the external field as well as by the state of the immediate neighborhood of the spin. The transition probabilities ω form a multidimensional matrix, which obeys the following restrictions. The first is that the sum of all the elements in a given column is zero. The second is that for each row the elements multiplied by the appropriate Boltzmann factor should add up to zero. The last criterion expresses the fact that the rate should be equal to zero when the system is in equilibrium.

We will now assume that one is dealing with ‘‘weak coupling,’’ that is to say, that the transition probabilities that are dependent on the magnetic fields and quadrupole field of the nearest neighbors can be expressed as a linear function of these two fields. The transition probability ω can be written as

$$\omega(S_k;S_k',S_{k-1},S_{k+1})=\omega(S_k,S_k')[1+m(S_{k-1}+S_{k+1})+d(S_{k-1}^2+S_{k+1}^2)], \tag{6}$$

where m and d stand for $\mu M/kT$ and D/kT, respectively.

The master equation can now be simplified using Eqs. (5) and (6):

$$\begin{aligned}\frac{dP(S_k)}{dt}=&\sum_{S_k'} \omega(S_k,S_k')P(S_k')\\&+m\sum_{S_{k-1},S_k'} \omega(S_k,S_k')S_{k-1}P(S_{k-1},S_k')\\&+m\sum_{S_k',S_{k+1}} \omega(S_k,S_k')S_{k+1}P(S_k',S_{k+1})\\&+d\sum_{S_{k-1},S_k'} \omega(S_k,S_k')S_{k-1}^2P(S_{k-1},S_k')\\&+d\sum_{S_k',S_{k+1}} \omega(S_k,S_k')S_{k+1}^2P(S_k',S_{k+1}).\end{aligned} \tag{7}$$

The condition

$$\sum_{S_k} P(S_k)=1 \tag{8}$$

leads to

$$\sum_{S_k} \omega(S_k,S_k')=0. \tag{9}$$

Furthermore, we have the equilibrium conditions

$$\sum_{S_k'} \omega(S_k,S_k')P_\infty(S_k')=0, \tag{10}$$

$$\sum_{S_{k-1},S_k'} \omega(S_k,S_k')S_{k-1}P_\infty(S_{k-1},S_k')=0, \tag{11}$$

$$\sum_{S_{k-1},S_k'} \omega(S_k,S_k')S_{k-1}^2P_\infty(S_{k-1},S_k')=0, \tag{12}$$

Equation (9) leads to, after evaluation,

$$\omega(++)+\omega(0+)+\omega(-+)=0, \tag{13a}$$

$$\omega(+0)+\omega(00)+\omega(-0)=0, \tag{13b}$$

$$\omega(+-)+\omega(0-)+\omega(--)=0. \tag{13c}$$

The equilibrium values for the probabilities are easily found, using Eq. (2):

$$P_\infty(+)=Z^{-1}e^{\beta+\gamma}, \tag{14a}$$

$$P_\infty(0)=Z^{-1}, \tag{14b}$$

$$P_\infty(-)=Z^{-1}e^{-\beta+\gamma}, \tag{14c}$$

with the partition function defined by

$$Z=e^{\beta+\gamma}+1+e^{-\beta+\gamma},$$

where $\beta=\mu H_s/kT$ and $\gamma=\Delta/kT$. Equation (10) leads to three more constraints on the ω's:

$$\omega(++)P_\infty(+)+\omega(+0)P_\infty(0)+\omega(+-)P_\infty(-)=0, \tag{15a}$$

$$\omega(0+)P_\infty(+)+\omega(00)P_\infty(0)+\omega(0-)P_\infty(-)=0, \tag{15b}$$

$$\omega(-+)P_\infty(+)+\omega(-0)P_\infty(0)+\omega(--)P_\infty(-)=0. \tag{15c}$$

These conditions are fulfilled by the following set of relations for the off-diagonal elements of the ω's:

$$\omega(+0)=\nu_+e^{\beta+\gamma}, \tag{16a}$$

$$\omega(0+)=\nu_+, \tag{16b}$$

$$\omega(-+)=\nu_2e^{-\beta}, \tag{16c}$$

$$\omega(+-)=\nu_2e^{\beta}, \tag{16d}$$

$$\omega(0-)=\nu_-, \tag{16e}$$

$$\omega(-0)=\nu_-e^{-\beta+\gamma}. \tag{16f}$$

The diagonal elements are determined by Eq. (13). From Eq. (11) one finds

$$\omega(++)[P_\infty(++)-P_\infty(-+)]+\omega(+0)[P_\infty(+0) -P_\infty(-0)]+\omega(+-)[P_\infty(+-)-P_\infty(--)]=0, \tag{17}$$

and two similar equations can be established by changing the first label. Also from Eq. (12)

$$\omega(++)[P_\infty(++)+P_\infty(-+)]+\omega(+0)[P_\infty(+0) +P_\infty(-0)]+\omega(+-)[P_\infty(+-)+P_\infty(--)]=0, \tag{18}$$

and its counterparts are found. From Eq. (14) we have

$$\langle S\rangle_\infty=\frac{2\sinh\beta}{e^{-\gamma}+2\cosh\beta}, \tag{19a}$$

$$\langle S^2\rangle_\infty=\frac{2\cosh\beta}{e^{-\gamma}+2\cosh\beta}\equiv\langle Q\rangle_\infty. \tag{19b}$$

We define here $Q=S^2$, as was used by Blume, Emery, and Griffith [1] and by Lajzerowicz and Sivardière [2,3]. This is different from the definition $Q=3S^2-2$ used by Chen and Levy [7] and Keskin and co-workers [13,25,26]. The last definition ensures that $Q=0$ at infinite temperature.

III. UNCOUPLED SPIN-1 ISING CHAIN

It may be helpful to begin our discussion of time-dependent processes with an extremely simple case: a single spin-1 Ising particle. We allow the particle to interact with a heat reservoir that induces spontaneous flips among the values $S_k=+1$, 0, and -1, with a given transition probability per unit time ω. If one makes no additional assumptions about the transition probabilities, i.e., using Eq. (16), one can see that the relaxations of $\langle S_k\rangle$ and $\langle Q_k\rangle$ are, in general, not independent of each other. However, in this paper we will make a special assumption about the rate constant, which we will choose in such a way that $\langle S_k\rangle$ returns to the equilibrium value $\langle S_\infty\rangle$. In this case the evaluation of the relaxation equation of the individual spins shows that $\langle S_k\rangle$ and $\langle Q_k\rangle$ relax separately, i.e., independently of each other. However, we will see that this is no longer the case in the weak-coupling approximation. The appearance of the temperature in the nonequilibrium equation is due to the requirement that the master equation must fulfill detailed balancing. In this model the relaxation time is always finite since the model does not contain any cooperative transitions, which usually give rise to infinite relaxation at the critical temperature.

IV. WEAKLY COUPLED SPIN-1 ISING CHAIN

In the spin-1 model we are dealing with eight averages. These averages can be expressed as linear combination of the joint probabilities as

$$\langle S_k\rangle=P(++)+P(+0)+P(+-)-P(-+) -P(-0)-P(--),$$

$$\langle Q_k\rangle=P(++)+P(+0)+P(+-)+P(-+)+P(-0) +P(--),$$

$$\langle S_{k+1}\rangle=P(++)+P(0+)+P(-+)-P(+-)-P(0-) -P(--),$$

$$\langle Q_{k+1}\rangle=P(++)+P(0+)+P(-+)+P(+-)+P(0-) +P(--),$$

$$\langle S_kS_{k+1}\rangle = P(++)-P(+-)-P(-+)+P(--), \tag{20}$$

$$\langle Q_kQ_{k+1}\rangle = P(++)+P(+-)+P(-+)+P(--),$$

$$\langle S_kQ_{k+1}\rangle = P(++)+P(+-)-P(-+)-P(--),$$

$$\langle Q_kS_{k+1}\rangle = P(++)-P(+-)+P(-+)-P(--),$$

$$1=\sum_{S_kS_{k+1}} P(S_kS_{k+1}).$$

The inverse of these equations determines the P's as linear combinations of these averages and the number 1, which appears only in the $P(00)$. From Eqs. (17)–(19) one obtains

$$\langle S_kS_{k+1}\rangle_\infty = \langle S\rangle_\infty^2 = S_\infty^2 = Z^{-2}4\sinh^2\beta,$$

$$\langle S_kQ_{k+1}\rangle_\infty = \langle S\rangle_\infty\langle Q\rangle_\infty = S_\infty Q_\infty = Z^{-2}4\sinh\beta\cosh\beta, \tag{21}$$

$$\langle Q_kQ_{k+1}\rangle_\infty = \langle Q\rangle_\infty^2 = Q_\infty^2 = Z^{-2}4\cosh^2\beta.$$

The general result of these calculations for $\langle S_k\rangle$ and $\langle Q_k\rangle$ is given by

$$\begin{aligned}\frac{d\langle S_k\rangle}{dt} &= a_1\langle S_k\rangle + a_2\langle Q_k\rangle + a_3 + a_1(m\langle S_{k-1}S_k\rangle + d\langle Q_{k-1}S_k\rangle \\ &\quad + m\langle S_kS_{k+1}\rangle + d\langle S_kQ_{k+1}\rangle) + a_2(m\langle S_{k-1}Q_k\rangle \\ &\quad + d\langle Q_{k-1}Q_k\rangle + m\langle Q_kS_{k+1}\rangle + d\langle Q_kQ_{k+1}\rangle) \\ &\quad + a_3(m\langle S_{k-1}\rangle + d\langle Q_{k-1}\rangle + m\langle S_{k+1}\rangle + d\langle Q_{k+1}\rangle),\end{aligned} \tag{22a}$$

$$\begin{aligned}\frac{d\langle Q_k\rangle}{dt} &= b_1\langle S_k\rangle + b_2\langle Q_k\rangle + b_3 + b_1(m\langle S_{k-1}S_k\rangle + d\langle Q_{k-1}S_k\rangle \\ &\quad + m\langle S_kS_{k+1}\rangle + d\langle S_kQ_{k+1}\rangle) + b_2(m\langle S_{k-1}Q_k\rangle \\ &\quad + d\langle Q_{k-1}Q_k\rangle + m\langle Q_kS_{k+1}\rangle + d\langle Q_kQ_{k+1}\rangle) \\ &\quad + b_3(m\langle S_{k-1}\rangle + d\langle Q_{k-1}\rangle + m\langle S_{k+1}\rangle + d\langle Q_{k+1}\rangle),\end{aligned} \tag{22b}$$

where

$$a_1 = -1/2(\nu_+ + \nu_-) - 2\nu_2\cosh\beta,$$

$$a_2 = -1/2(\nu_+ - \nu_-) - 2\nu_2\sinh\beta - (\nu_+e^{\beta+\gamma} - \nu_-e^{-\beta+\gamma}),$$

$$a_3 = \nu + e^{\beta+\gamma} - \nu_-e^{-\beta+\gamma},$$

$$b_1 = -1/2(\nu_+ - \nu_-),$$

$$b_2 = -1/2(\nu_+ + \nu_-) - (\nu_+e^{\beta+\gamma} + \nu_-e^{-\beta+\gamma}),$$

$$b_3 = \nu + e^{\beta+\gamma} + \nu_-e^{-\beta+\gamma}.$$

We now make a special assumption about the rate constants ν, i.e., $\nu_+ = \nu_- = \nu$ and $\nu_2 = \nu e^{\gamma}$. Here we are guided by the necessity that $\langle S_k\rangle$ should return to the equilibrium value $\langle S_k\rangle_\infty$. After this modification we obtain

$$\begin{aligned}\frac{1}{\nu Z}\frac{d\langle S_k\rangle}{dt} &= -(\langle S_k\rangle - \langle S_k\rangle_\infty) - m(\langle S_kS_{k-1}\rangle - \langle S_k\rangle_\infty\langle S_{k-1}\rangle) \\ &\quad - d(\langle S_kQ_{k-1}\rangle - \langle S_k\rangle_\infty\langle Q_{k-1}\rangle) - m(\langle S_kS_{k+1}\rangle \\ &\quad - \langle S_k\rangle_\infty\langle S_{k+1}\rangle) - d(\langle S_kQ_{k+1}\rangle - \langle S_k\rangle_\infty\langle Q_{k+1}\rangle),\end{aligned} \tag{23a}$$

$$\begin{aligned}\frac{1}{\nu Z}\frac{d\langle Q_k\rangle}{dt} &= -(\langle Q_k\rangle - \langle Q_k\rangle_\infty) - m(\langle Q_kS_{k-1}\rangle \\ &\quad - \langle Q_k\rangle_\infty\langle S_{k-1}\rangle) - d(\langle Q_kQ_{k-1}\rangle \\ &\quad - \langle Q_k\rangle_\infty\langle Q_{k-1}\rangle) - m(\langle Q_kS_{k+1}\rangle \\ &\quad - \langle Q_k\rangle_\infty\langle S_{k+1}\rangle) - d(\langle Q_kQ_{k+1}\rangle \\ &\quad - \langle Q_k\rangle_\infty\langle Q_{k+1}\rangle).\end{aligned} \tag{23b}$$

These equations can be simplified if one introduces the relative deviations from the equilibrium

$$X_k^S = \langle S_k\rangle - S_\infty, \tag{24a}$$

$$X_k^Q = \langle Q_k\rangle - Q_\infty. \tag{24b}$$

which lead to

$$\begin{aligned}\frac{1}{\nu Z}\frac{dX_k^S}{dt} &= -X_k^S - m(Y_{k-1}^{SS} - S_\infty X_{k-1}^S) - d(Y_{k-1}^{QS} - S_\infty X_{k-1}^Q) \\ &\quad - m(Y_k^{SS} - S_\infty X_{k+1}^S) - d(Y_k^{QS} - S_\infty X_{k+1}^Q),\end{aligned} \tag{25a}$$

$$\begin{aligned}\frac{1}{\nu Z}\frac{dX_k^Q}{dt} &= -X_k^Q - m(Y_{k-1}^{SQ} - Q_\infty X_{k-1}^S) - d(Y_{k-1}^{QQ} - Q_\infty X_{k-1}^Q) \\ &\quad - m(Y_k^{QS} - Q_\infty X_{k+1}^S) - d(Y_k^{QQ} - Q_\infty X_{k+1}^Q).\end{aligned} \tag{25b}$$

Similar equations can be found for the coupling averages using in the master equation for the pair distribution function; see Appendix. The result is

$$\frac{1}{\nu Z}\frac{dY_k^{SS}}{dt} = -2Y_k^{SS} + S_\infty X_k^S + S_\infty X_{k+1}^S, \tag{25c}$$

$$\frac{1}{\nu Z}\frac{dY_k^{SQ}}{dt} = -2Y_k^{SQ} + Q_\infty X_k^S + S_\infty X_{k+1}^Q, \tag{25d}$$

$$\frac{1}{\nu Z}\frac{dY_k^{QS}}{dt} = -2Y_k^{QS} + S_\infty X_k^Q + Q_\infty X_{k+1}^S, \tag{25e}$$

$$\frac{1}{\nu Z}\frac{dY_k^{QQ}}{dt} = -2Y_k^{QQ} + Q_\infty X_k^Q + Q_\infty X_{k+1}^Q, \tag{25f}$$

where

$$Y_k^{SS} = \langle S_kS_{k+1}\rangle - S_\infty{}^2, \tag{26a}$$

$$Y_k^{SQ} = \langle S_kQ_{k+1}\rangle - S_\infty Q_\infty, \tag{26b}$$

$$Y_k^{QS} = \langle Q_kS_{k+1}\rangle - S_\infty Q_\infty, \tag{26c}$$

$$Y_k^{QQ}=\langle Q_k Q_{k+1}\rangle - Q_\infty{}^2. \tag{26d}$$

These rate equations form the starting point for the solution of the problem.

Equations (25) form a set of N coupled equations in $X_k^S, X_k^Q, Y_k^{SS}, Y_k^{SQ}, Y_k^{QS}, Y_k^{QQ}$. They can be solved by using a Fourier transform, which automatically satisfies the boundary condition that spins 0 and N are equivalent. Hence we introduce

$$\begin{pmatrix} X_k^S \\ X_k^Q \\ Y_k^{SS} \\ Y_k^{SQ} \\ Y_k^{QS} \\ Y_k^{QQ} \end{pmatrix} = \frac{1}{\sqrt{N}} \sum \begin{pmatrix} a_l^{(1)} \\ a_l^{(2)} \\ a_l^{(3)} \\ a_l^{(4)} \\ a_l^{(5)} \\ a_l^{(6)} \end{pmatrix} e^{2\pi i l k/N} \tag{27}$$

and

$$a_l^{(i)}(t)=a_l^{(i)}(0)e^{-t/\tau(l)} \quad (i=1,\ldots,6), \tag{28}$$

where τ depends on l, the Fourier component. The equations of motion

$$\frac{da_l^{(i)}}{dt}=Z\nu \sum_j M_{ij} a_l^{(j)}, \tag{29}$$

using the matrix $\mathbf{M}$,

$$\mathbf{M}=\begin{pmatrix} -1+mC_1S_\infty & dC_1S_\infty & -mC_2 & -d & -dC_3 & 0 \\ mC_1Q_\infty & -1+dC_1Q_\infty & 0 & -mC_3 & -m & -dC_2 \\ C_2^*S_\infty & 0 & -2 & 0 & 0 & 0 \\ Q_\infty & C_3^*S_\infty & 0 & -2 & 0 & 0 \\ C_3^*Q_\infty & S_\infty & 0 & 0 & -2 & 0 \\ 0 & C_2^*Q_\infty & 0 & 0 & 0 & -2 \end{pmatrix}, \tag{30}$$

and using $C_1=2\cos 2\pi l/N$, $C_2=2e^{-\pi l i/N}\cos \pi l/N$, and $C_3=e^{-2\pi l i/N}$ and their complex conjugates.

To find the relaxation times requires the solution of the secular determinant

$$\mathrm{Det}(\lambda \mathbf{1}-\mathbf{M})=0, \tag{31}$$

with $\lambda=-1/Z\nu\tau$. Ignoring the correlations, i.e., using only the four elements in the left upper corner of the determinant, we find two solutions

$$\frac{1}{\tau_1}=Z\nu, \tag{32a}$$

$$\frac{1}{\tau_2}=Z\nu(1-mC_1S_\infty-dC_1Q_\infty). \tag{32b}$$

It is interesting to notice that the relaxations of $\langle S_k\rangle$ and $\langle Q_k\rangle$ are now coupled even in zeroth order. In order to diagonalize this submatrix, we introduce the right and left eigenvectors

$$[T^L]=N_{rm}\begin{bmatrix} -C_1Q_\infty & C_1S_\infty \\ m & d \end{bmatrix}, \tag{33a}$$

$$[T^R]=N_{rm}\begin{bmatrix} d & C_1S_\infty \\ m & C_1Q_\infty \end{bmatrix}, \tag{33b}$$

with the norm N_{rm} given by $(mC_1S_\infty+dC_1Q_\infty)^{-1/2}$. This transformation is now applied to the 6×6 matrix given by Eq. (30). As a result of this, the matrix elements $M_{13},\ldots,M_{16}$ and $M_{23},\ldots,M_{26}$ will be replaced by linear combinations. Similarly, the matrix elemements $M_{31},\ldots,M_{61}$ and $M_{32},\ldots,M_{62}$ will be replaced by linear combinations, using the transformation matices T^R and T^R. A first-order perturbation calculation was applied to this new matrix. This result in the corrected eigenvalues

$$\frac{1}{\tau_1}=Z\nu\left(1+2\,\frac{m^2S_\infty^2-2mdS_\infty Q_\infty\cos 2\pi l/N+d^2Q_\infty^2}{mS_\infty+dQ_\infty}\right), \tag{34a}$$

$$\frac{1}{\tau_2}=Z\nu\left(1-2(mS_\infty+dQ_\infty)\cos 2\pi l/N+4\,\frac{m^2S_\infty^2\cos 2\pi l/N-2mdS_\infty Q_\infty(1+\cos 2\pi l/N)+d^2Q_\infty^2\cos 2\pi l/N}{mS_\infty+dQ_\infty}\right), \tag{34b}$$

and

$$\frac{1}{\tau_3}=2Z\nu. \tag{34c}$$

The last relaxation time is the uncorrected value, given for reference only. The discussion of these results will be given in the next section.

V. SUMMARY AND DISCUSSION

In this paper, we have studied the time-dependent one-dimensional spin-1 Ising system by means of the modified version of Glauber's one-dimensional spin relaxation model and the resulting set of rate equations for the average values of dipole and quadrupole moments, as well as their coupling. The results are found by using the following assumptions. (i) The transition probabilities for any spin-1 Ising particle depends on the state of the neighboring spin-1 Ising particles. This dependence is determined, in part, by the detailed balancing condition obeyed by the equilibrium state of the system. (ii) The rate constants are chosen in such a way that the average values of the dipole moment will return to the equilibrium value. (iii) The Ising interaction between the spin-1 particles is assumed to be weak compared to the coupling with the heat bath and in this way we obtain a cutoff after a finite set of equations. The rate equations are solved by using a Fourier transform, which automatically satisfies the boundary condition that spins 0 and N are identical.

Finally, we determine the two relaxation times by using a perturbation method. The secular matrix of the problem contains a 2×2 subspace, which, after diagonalization, leads to two slightly different relaxation times τ_1 and τ_2. These two relaxation times are further altered due to the presence of the double relaxation times of the pair function. This modification is worked out using first-order perturbation theory.

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APPENDIX: MASTER EQUATION FOR THE PAIR DISTRIBUTION FUNCTION

The master equation for the pair distribution function (also called the time derivative of the two-spin distribution function) is given by

$$\begin{aligned}\frac{dP(S_k,S_{k+1})}{dt}&=\sum_{S_k',S_{k+1}'}\omega(S_k,S_{k+1};S_k',S_{k+1}')P(S_k',S_{k+1}')\\&=\sum_{S_k',S_{k+1}'}\omega(S_k,S_k')\,\delta(S_{k+1}-S_{k+1}')\\&\quad\times P(S_k',S_{k+1}')+\sum_{S_k',S_{k+1}'}\omega(S_{k+1},S_{k+1}')\\&\quad\times\delta(S_k-S_k')P(S_k',S_{k+1}'),\end{aligned} \tag{A1}$$

$$\begin{aligned}\frac{dP(S_k,S_{k+1})}{dt}&=\sum_{S_k'}\omega(S_k;S_k')P(S_k',S_{k+1})\\&\quad+\sum_{S_{k+1}'}\omega(S_{k+1};S_{k+1}')P(S_k,S_{k+1}').\end{aligned} \tag{A2}$$

Here two assumptions are made: (i) the function depends parametrically on the state of the neighbors and (ii) the function depends linearly on the neighboring spins. However, the terms of higher order in the coupling constant are neglected, i.e., we assume that m and d are very small. This equation can easily be modified for the joint probability functions $P(Q_k,Q_{k+1})$, $P(S_k,S_{k+1})$, etc.

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